Integrals of Four Variables with Statistical Distribution Associated with Hyper Geometric Function of Matrix Argument

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INTRODUCTION

In this paper introduce matrix sequence, matrix series and concepts analog to convergence of series in scales variable. a matrix series is obtained by adding up the matrices in a matrix sequence for example if $A_{0,}A_{1\,,}A_2$………………is a matrix series given by

F(A) = $\sum_{k=0}^{\infty} A_k$…………………………………………………. (1.1.1)

If the matrix series is a power series, then we will be considering powers of matrices and hence in this case the series will be defined only for n x n matrices, for an n x n matrix A. consider the series.

g(A) = $a_{0,}$I+ $a_{1\,,}$A +……..+$a_{k,}A^k$ +…….=
$\sum_{k=0}^{\infty}$ $a_{k,}\, A_k$ ……………………………………………………(1.1.2)

where $a_{0,}, a_{1\,,}$…………..$a_{k,}$ are scalars.

As in the case of scalars series, convergence of matrix will be defined in terms of the sums.

A general hypergeometric series $pF_{q,}$ (.) in a real scalar variable X is defined as follow.

$pF_q\ \ (a_{1\,,} \ldots \ldots \ldots \ldots \ldots . . a_{p\,;}\ \ b_1\ldots\ldots, b_{q;\,,} X)\ =\ \ \sum_{k=0}^{\infty} \frac{(a_{1\,,})_{\,r} \ldots\ldots (a_{p\,,})_r\, X^r}{(b_{1\,,})_{\,r} \ldots\ldots (b_{p\,,})_r\ r\,!}$
……………………………………………………………………(2,.1.3)

For $(a)_m$ = a(a+1)……(a+m-1) $(a)_0$ = 1 a ≠ 0

For example oF_0 (; ; X) =e^x

$1F_0$ (α; ; X) =$(I-X)^{-\alpha}$ for |X|<1

In (1.1.3) there are p upper parameter $a_{1\,,} \ldots \ldots \ldots a_{p\,;}$ and q lower parameters $b_{1 \ldots\ldots\ldots\ldots\ldots}, b_{q;\,,}$. The series in (1.1.3) is convergent for all X if q ≥ p convergent for $|X|$<1 if p= q+1 divergent if P > q+1 and the convergence. Condition for X=1 and X = -1 can also be work out. a matrix series in an n X n matrix. A corresponding to the right side in (1.1.3) is obtain by replacing X by A, thus we may define a hypergeometric series in an n X n matrix. A as follows.

$$pF_q\ (a_{1\,,} \ldots . a_{p\,;}\ \ b_{1 \ldots\ldots}, b_{q;\,,} A) = \text{q} \geq \text{p}\ \ \sum_{r=0}^{\infty} \frac{(a_{1\,,})_{\ r} \ldots\ldots\ldots\ldots (a_{p\,,})_r\ X^r}{(b_{1\,,})_{\ r} \ldots\ldots\ldots\ldots (b_{p\,,})_r\ r\,!}$$

……………………………………………………………………..(1.1.4)

where $a_{1\,,} \ldots\ldots\ldots\ldots . a_{p\,,}\ b_{1 \ldots\ldots\ldots\ldots\ldots}, b_{q;\,,}$ are scalars. The series in (1.1.4) is convergent for all A if q ≥ p convergent for p= q+1 when the Eigen values of A are less than 1 in absolute value and divergent when p>q+1 similarly it may be defined for two, three and four variable. Other definition involving in this paper for four variable of m X m matrix. Exactly similar analogous in scalar variable due to Exton (1995) . in what follow we shall take p, q, r and s to be positive integer of the symbol X and Δ(n ,a) stand the sequence of parameters of square positive definite matrices $X_{1\,,} X_{2\,,} \ldots \ldots \ldots \ldots \ldots . . X_r$ of order n x n and $\frac{\alpha}{n}$, $\frac{\alpha+1}{n} \ldots \ldots \ldots \ldots . . \frac{\alpha+n-1}{n}$ respectively. Also in all be established here after ,proper conditions of convergence of the involved are assumed. In which follows X,Y,Z,T,U, etc . matrices are positive definite symmetric of same order m x m.

1.1 Matrix – Variant real gamma density

The matrix – variant gamma density is defined as follow let X =X'>0 be a real matrix random variable then

$$\text{f(X)} = \frac{|X|^{\alpha - \frac{p+1}{2}}\ e^{-tr(X)}}{\Gamma_p(\alpha)}\ , \text{X= X'>0}$$

for Re(α)> P-1 or f(X) =0 elsewhere and also $\int_{x>0} f(x)dx$ =1 is known as the matrix – variant real gamma density .

1.2 INTEGRALS OF FOUR VARIABLE WITH STATISTICAL DISTIBUTION ASSOCIAT WITH HYPERGEOMETRIC FUNCTION OF MATRIX ARGUMENT

The formula to be establish are as follow

[1] $\int_0^I U^{a-\frac{m+1}{2}} \ (I-U)^{b-\frac{m+1}{2}} \ F_1$ (a,1-a;c;$\frac{U}{2}$).

$F_1{}^4\ [a_{1,}, a_{1,}, a_{1,}, a_{2,} b_{1,} b_{1,}, b_{1,} b_{1,} c_{1,} c_{2,} c_{3,} c_{4;,}$ X $(I-U)^n$, Y$(I-U)^n$, Z $(I-U)^n$, T $(I-U)^n$]dU

$$=\mathrm{L}F_1{}^4\begin{bmatrix} a_{1,}, a_{1,}, a_{1,}, a_{2,} b_{1,} b_{1,}, b_{1,} b_{1,} c_{1,} c_{2,} c_{3,} c_{4;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}) \\ \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); \mathrm{X}; \mathrm{Y}; \mathrm{Z}; \mathrm{T} \end{bmatrix}$$

…………………………………………………………………….(2.1.1)

Where

$F_1{}^4\ [a_{1,}, a_{1,}, a_{1,}, a_{2,} b_{1,} b_{1,}, b_{1,} b_{1,} c_{1,} c_{2,} c_{3,} c_{4;,}\ (I-U)^n$, X$(I-U)^n$, Y $(I-U)^n$, Z $(I-U)^n$T]

$$=\sum_{p,q,r,s=0}^{\infty} \frac{(a_{1,})_{p+q+r}\ (a_{1,})_s\ (b_1)_{p+q+r+s}}{p!s!q!r!(c_{1,})_p\ (C_{2,})_q\ (C_{3,})_r\ (C_{4,})_s}\ (I-U)^{np}\ X^p\ (I-U)^{nq}\ Y^q\ (I-U)^{nr} Z^r\ (I-U)^{ns} T^s$$

So that

$F_1{}^4\ [a_{1,}, a_{1,}, a_{1,}, a_{2,} b_{1,} b_{1,}, b_{1,} b_{1,} c_{1,} c_{2,} c_{3,} c_{4;,}\ (I-U)^n$, X$(I-U)^n$, Y $(I-U)^n$, Z $(I-U)^n$T]

$= F_1{}^4$ [$(I-U)^n$, X$(I-U)^n$, Y $(I-U)^n$, Z $(I-U)^n$T

Then a probability density function (p.d.f) of (2.1.1) is given by :

$$\mathrm{F(U)} = \frac{U^{a-\frac{p+1}{2}}\ (I-U)^{b-\frac{p+1}{2}}\ F_1(X_1) F_1{}^4\ [X_2]}{LF_1{}^4\ [X_3]}$$

=0 elsewhere

Where

$X_1 =$ (a,a-1 ;c; $\frac{U}{2}$)

$X_2 =$ [X$(I-U)^n$, Y $(I-U)^n$, Z $(I-U)^n$, $(I-U)^n$T]

$$X_3=\begin{bmatrix} a_{1,}, a_{1,}, a_{2,}, a_{1,} b_{1,} b_{1,}, b_{1,} b_{1,} c_{1,} c_{2,} c_{3,} c_{4;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}) \\ , \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{bmatrix}$$

[2] $\int_0^I U^{a-\frac{m+1}{2}} \ (I-U)^{b-\frac{m+1}{2}} \ F_1$ (a,1-a;c;$\frac{U}{2}$).

${F_2}^4$ [$(I-U)^n$ X,$(I-U)^n$ Y, $(I-U)^n$ Z, $(I-U)^n$T]dU

$$= L{F_2}^4 \begin{bmatrix} a_{1,}, a_{1,}, a_{2,}, a_{2,} b_{1,} b_{1,}, b_{1,} b_{1,} c_{1,} c_{2,} c_{3,} c_{4;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}) \\ \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{bmatrix}$$

……………………………..……………………………………..(1.2.2)

Where

${F_2}^4$ [$a_{1,}, a_{1,}, a_{2,}, a_{2,} b_{1,} b_{1,}, b_{1,} b_{1,} c_{1,} c_{2,} c_{3,} c_{4;,}$ $(I-U)^n$ X,$(I-U)^n$ Y, $(I-U)^n$ Z, $(I-U)^n$T]

$$=\sum_{p,q,r,s=0}^{\infty} \frac{(a_{1,})_{p+q} (a_{1,})_{r+s} (b_1)_{p+q+r+s}}{p!s!q!r!(c_{1,})_p (C_{2,})_q (C_{3,})_r (C_{4,})_s} \ (I-U)^{np} X^p \ (I-U)^{nq} \ Y^q \ (I-U)^{nr} Z^r \ (I-U)^{ns} T^s$$

Then probability density function (p.d.f) of (1.2.2) is given by :

$$F(U) = \frac{U^{a-\frac{m+1}{2}} \ (I-U)^{b-\frac{m+1}{2}} \ F_1(X_1) {F_2}^4 \ [X_2]}{L{F_2}^4 \ [X_4]}$$

=0 elsewhere

Where

$$X_4=\begin{bmatrix} a_{1,}, a_{1,}, a_{2,}, a_{2,} b_{1,} b_{1,}, b_{1,} b_{1,} c_{1,} c_{2,} c_{3,} c_{4;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}) \\ (\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{bmatrix}$$

[3] $\int_0^I U^{a-\frac{m+1}{2}} \ (I-U)^{b-\frac{m+1}{2}} \ F_1$ (a,1-a;c;$\frac{U}{2}$).

${F_3}^4$ [$(I-U)^n$ X,$(I-U)^n$ Y, $(I-U)^n$ Z, $(I-U)^n$T]dU

$$= LF_3{}^4 \begin{bmatrix} a_{1,}, a_{1,}, a_{2,}, a_{3,} b_{1,} b_{1,}, b_{1,} b_{1,} c_{1,} c_{2,} c_{3,} c_{4;}, \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}) \\ \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{bmatrix}$$

………………………………………………………………..………..(1.1.3)

Where s

$F_3{}^4\ [a_{1,}, a_{1,}, a_{2,}, a_{3,} b_{1,} b_{1,}, b_{1,} b_{1,} c_{1,} c_{2,} c_{3,} c_{4;}, (I-U)^n$ X,$(I-U)^n$ Y, $(I-U)^n$ Z, $(I-U)^n$T]

$$=\sum_{p,q,r,s=0}^{\infty} \frac{(a_{1,})_{p+q}\,(a_{2,})_r\,(a_{3,})_r\,(b_1)_{p+q+r+s}}{p!s!q!r!(c_{1,})_p\,(c_{2,})_q\,(C_{3,})_r\,(C_{4,})_s} \ (I-U)^{np} X^p\,(I-U)^{nq}\,Y^q\,(I-U)^{nr} Z^r\,(I-U)^{ns} T^s$$

Then probability density function (p.d.f) of (1.1.3) is given by :

$$F(U) = \frac{U^{a-\frac{m+1}{2}}\ (I-U)^{b-\frac{m+1}{2}}\ F_1(X_1) F_3{}^4\,[X_2]}{LF_3{}^4\,[X_5]}$$

=0 elsewhere

Where

$$X_5 = \begin{bmatrix} a_{1,}, a_{1,}, a_{2,}, a_{3,} b_{1,} b_{1,}, b_{1,} b_{1,} c_{1,} c_{2,} c_{3,} c_{4;}, \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}) \\ \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{bmatrix}$$

[4] $\int_0^I U^{a-\frac{m+1}{2}}\ (I-U)^{b-\frac{m+1}{2}}\ F_1$ (a,1-a;c;$\frac{U}{2}$).

$F_4{}^4\ [(I-U)^n$ X,$(I-U)^n$ Y, $(I-U)^n$ Z, $(I-U)^n$T]dU

$$= LF_4{}^4 \begin{bmatrix} a_{1,}, a_{1,}, a_{2,}, a_{3,} b_{1,} b_{1,}, b_{1,} b_{1,} c_{1,} c_{2,} c_{3,} c_{4;}, \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}) \\ \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{bmatrix}$$

………………………………………………………………………(1.1.4)

Where

$F_4{}^4\ [a_{1,}, a_{1,}, a_{1,}, a_{1,} b_{1,} b_{1,}, b_{1,} b_{1,} c_{1,} c_{2,} c_{3,} c_{4;}, X, Y, Z, T]$

$$=\sum_{p,q,r,s=0}^{\infty} \frac{(a_{1,})_{p+q+r+s}(b_{1,})_{p+q+rs}(b_{2,})_r}{p!s!q!r!(c_{1,})_p\,(c_{2,})_q\,(C_{3,})_r\,(C_{4,})_s} \ X^p\ Y^q\,Z^r\,T^s$$

Then probability density function (p.d.f) of (1.1.4) is given by :

$$F(U) = \frac{U^{a-\frac{m+1}{2}}\ (I-U)^{b-\frac{m+1}{2}}\ F_1(X_1)F_4{}^4\,[X_2]}{LF_4{}^4\,[X_6]}$$

=0 elsewhere

Where

$$X_6=\begin{bmatrix} a_{1\,,}, a_{1\,,}, a_{1,}, a_{1\,,} b_{1\,,} b_{1\,,}, b_{2\,,} b_{1\,,} c_{1\,,} c_{2\,,} c_{3\,,} c_{4;\,,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}) \\ \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{bmatrix}$$

[5] $\int_0^I U^{a-\frac{m+1}{2}}\ (I-U)^{b-\frac{m+1}{2}}\ F_1$ (a,1-a;c;$\frac{U}{2}$).

$F_5{}^4\ [(I-U)^n$ X,$(I-U)^n$ Y, $(I-U)^n$ Z, $(I-U)^n$T]dU

=

$$LF_5{}^4\begin{bmatrix} a_{1\,,}, a_{1\,,}, a_{1\,,}, a_{2\,,} b_{1\,,} b_{1\,,}, b_{2\,,} b_{2\,,} c_{1\,,} c_{2\,,} c_{3\,,} c_{4;\,,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}) \\ \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{bmatrix}$$…………………

…….……………………………………………..(1.1.5)

Where

$F_5{}^4\ [a_{1\,,}, a_{1\,,}, a_{1\,,}, a_{2\,,} b_{1\,,} b_{1\,,}, b_{2\,,} b_{2\,,} c_{1\,,} c_{2\,,} c_{3\,,} c_{4;\,,} X, Y, Z, T]$

$$=\sum_{p,q,r,s=0}^{\infty} \frac{(a_{1\,,})_{p+q+r}(a_{2\,,})_s(b_{1\,,})_{p+q}(b_{2\,,})_{r+s}}{p!s!q!r!(c_{1,})_p\ (c_{2\,,})_q\ (c_{3\,,})_r\,(c_{4\,,})_s}\ \ X^p\ \ Y^q\ Z^r\ T^s$$

Then probability density function (p.d.f) of (1.1.5) is given by :

$$F(U) = \frac{U^{a-\frac{m+1}{2}}\ (I-U)^{b-\frac{m+1}{2}}\ F_1(X_1)F_5{}^4\,[X_2]}{LF_5{}^4\,[X_7]}$$

=0 elsewhere

Where

$$X_7=\begin{bmatrix} a_{1\,,}, a_{1\,,}, a_{1,}, a_{2\,,} b_{1\,,} b_{1\,,}, b_{2\,,} b_{2\,,} c_{1\,,} c_{2\,,} c_{3\,,} c_{4;\,,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}) \\ \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{bmatrix}$$

[6] $\int_0^I U^{a-\frac{m+1}{2}}\ (I-U)^{b-\frac{m+1}{2}}\ F_1$ (a,1-a;c;$\frac{U}{2}$).

${F_6}^4$ [$(I-U)^n$ X,$(I-U)^n$ Y, $(I-U)^n$ Z, $(I-U)^n$T]dU

=

$$L{F_6}^4\begin{bmatrix} a_{1,}, a_{1,}, a_{1,}, a_{2,} b_{1,} b_{2,}, b_{3,} b_{1,} c_{1,} c_{2,} c_{3,} c_{4;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}) \\ , \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{bmatrix} \ldots\ldots$$

……………………………………………………..(1.1.6)

Where

${F_6}^4$ $[a_{1,}, a_{1,}, a_{1,}, a_{2,} b_{1,} b_{2,}, b_{3,} b_{1,} c_{1,} c_{2,} c_{3,} c_{4;,} X, Y, Z, T]$

$=\sum_{p,q,r,s=0}^{\infty} \frac{(a_{1,})_{p+q+r}(a_{2,})_s (b_{1,})_{p+s} (b_{2,})_q (b_{3,})_s}{p!s!q!r!(c_{1,})_p\ (c_{2,})_q\ (c_{3,})_r\ (c_{4,})_s}\ X^p\ Y^q\ Z^r\ T^s$

Then probability density function (p.d.f) of (1.1.6) is given by :

$$F(U) = \frac{U^{a-\frac{m+1}{2}}\ (I-U)^{b-\frac{m+1}{2}}\ F_1(X_1){F_6}^4\ [X_2]}{L{F_6}^4\ [X_8]}$$

=0 elsewhere

Where

$$X_8 = \begin{bmatrix} a_{1,}, a_{1,}, a_{1,}, a_{2,} b_{1,} b_{2,}, b_{3,} b_{1,} c_{1,} c_{2,} c_{3,} c_{4;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}) \\ \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{bmatrix}$$

[7] $\int_0^I U^{a-\frac{m+1}{2}}\ (I-U)^{b-\frac{m+1}{2}}\ F_1$ (a,1-a;c;$\frac{U}{2}$).

${F_7}^4$ [$(I-U)^n$ X,$(I-U)^n$ Y, $(I-U)^n$ Z, $(I-U)^n$T]dU

=

$$L{F_7}^4\begin{bmatrix} a_{1,}, a_{1,}, a_{2,}, a_{2,} b_{1,} b_{2,}, b_{1,} b_{2,} c_{1,} c_{2,} c_{3,} c_{4;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}) \\ \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{bmatrix} \ldots\ldots$$

……………………………………………………..(1.1.7)

where

$F_7{}^4\ [a_{1\,,}, a_{1\,,}, a_{2\,,}, a_{2\,,} b_{1\,,} b_{2\,,}, b_{1\,,} b_{2\,,} c_{1\,,} c_{2\,,} c_{3\,,} c_{4;\,,} X, Y, Z, T]$

$=\sum_{p,q,r,s=0}^{\infty} \frac{(a_{1\,,})_{p+q}(a_{2\,,})_{r+s}(b_{1\,,})_{p+r}(b_{2\,,})_q(b_{3\,,})_s}{p!s!q!r!(c_{1,})_p\ (C_{2\,,})_q\ (C_{3\,,})_{r+s}(C_{4\,,})_s}\ \ X^p\ \ Y^q\ Z^r\ T^s$

Then probability density function (p.d.f) of (1.1.7) is given by :

$$F(U) = \frac{U^{a-\frac{m+1}{2}}\ \ (I-U)^{b-\frac{m+1}{2}}\ F_1(X_1)F_7{}^4\ [X_2]}{LF_7{}^4\ [X_9]}$$

=0 elsewhere

Where

$$X_9 = \begin{bmatrix} a_{1\,,}, a_{1\,,}, a_2\ \ , a_{2\,,} b_{1\,,} b_{2,}, b_{1,} b_{2\,,} c_{1\,,} c_{2\,,} c_{3\,,} c_{4;\,,} \Delta(n,b), \Delta(\frac{n}{2}\ , \frac{c+b}{2}) \\ , \Delta(\frac{n}{2}\ , \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}\ , \frac{1+c+b-a}{2}); \text{X; Y; Z; T} \end{bmatrix}$$

[8] $\int_0^I U^{a-\frac{m+1}{2}}\ \ (I-U)^{b-\frac{m+1}{2}}\ \ F_1$ (a,1-a;c;$\frac{U}{2}$).

$F_7{}^4\ [(I-U)^n$ X,$(I-U)^n$ Y, $(I-U)^n$ Z, $(I-U)^n$T]dU

=

$$LF_8{}^4 \begin{bmatrix} a_{1\,,}, a_{1\,,}, a_{2\,,}, a_{2\,,} b_{1\,,} b_{2\,,}, b_{1\,,} b_{3\,,} c_{1\,,} c_{2\,,} c_{3\,,} c_{4;\,,} \Delta(n,b), \Delta(\frac{n}{2}\ , \frac{c+b}{2}) \\ , \Delta(\frac{n}{2}\ , \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}\ , \frac{1+c+b-a}{2}); \text{X; Y; Z; T} \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots$$

………..…………………………………………......(1.1.8)

Where

$F_8{}^4\ [a_{1\,,}, a_{1\,,}, a_{2\,,}, a_{2\,,} b_{1\,,} b_{2\,,}, b_{1\,,} b_{3\,,} c_{1\,,} c_{2\,,} c_{3\,,} c_{4;\,,} X, Y, Z, T]$

$=\sum_{p,q,r,s=0}^{\infty} \frac{(a_{1\,,})_{p+q}(a_{2\,,})_{r+s}(b_{1\,,})_{p+r}(b_{2\,,})_q(b_{3\,,})_s}{p!s!q!r!(c_{1,})_p\ (C_{2\,,})_q\ (C_{3\,,})_{r(}C_{4\,,})_s}\ \ X^p\ \ Y^q\ Z^r\ T^s$

Then probability density function (p.d.f) of (1.1.8) is given by :

$$F(U) = \frac{U^{a-\frac{m+1}{2}}\ \ (I-U)^{b-\frac{m+1}{2}}\ F_1(X_1)F_8{}^4\ [X_2]}{LF_8{}^4\ [X_{10}]}$$

=0 elsewhere

Where

$$X_{10}=\begin{bmatrix} a_{1,}, a_{1,}, a_2, a_{2,} b_{1,} b_{2,}, b_{3,} b_{2,} c_{1,} c_{2,} c_{3,} c_{4;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}) \\ , \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{bmatrix}$$

[9] $\int_0^I U^{a-\frac{m+1}{2}}\ (I-U)^{b-\frac{m+1}{2}}\ F_1$ (a,1-a;c;$\frac{U}{2}$).

${F_8}^4$ [$(I-U)^n$ X,$(I-U)^n$ Y, $(I-U)^n$ Z, $(I-U)^n$T]dU

=

$$L{F_9}^4 \begin{bmatrix} a_{1,}, a_{1,}, a_{1,}, a_{2,} b_{1,} b_{1,}, b_{1,} b_{1,} c_{1,} c_{2,} c_{3,} c_{1;,} \Delta(n,b), \Delta\left(\frac{n}{2}, \frac{c+b}{2}\right) \\ , \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots$$

……………………………………………………..(1.1.9)

Where

${F_9}^4$ $[a_{1,}, a_{1,}, a_{1,}, a_{2,} b_{1,} b_{1,}, b_{1,} b_{1,} c_{1,} c_{2,} c_{3,} c_{1;,} X, Y, Z, T]$

$$=\sum_{p,q,r,s=0}^{\infty} \frac{(a_{1,})_{p+q+r} (a_{2,})_s (b_{1,})_{p+q+r+s}}{p!s!q!r!(c_{1,})_{p+s}\ (c_{2,})_q\ (c_{3,})_r}\ X^p\ Y^q\ Z^r\ T^s$$

Then probability density function (p.d.f) of (1.1.9) is given by :

$$F(U) = \frac{U^{a-\frac{m+1}{2}}\ (I-U)^{b-\frac{m+1}{2}}\ F_1(X_1) {F_9}^4\ [X_2]}{L{F_9}^4\ [X_{11}]}$$

=0 else where

Where

$$X_{11} = \begin{bmatrix} a_{1,}, a_{1,}, a_1, a_{2,} b_{1,} b_{1,}, b_{1,} b_{1,} c_{1,} c_{2,} c_{3,} c_{1;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \Delta(\frac{n}{2}, \frac{1+c+b}{2}) \\ \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{bmatrix}$$

[10] $\int_0^I U^{a-\frac{m+1}{2}}\ (I-U)^{b-\frac{m+1}{2}}\ F_1$ (a,1-a;c;$\frac{U}{2}$).

${F_{10}}^4$ [$(I-U)^n$ X,$(I-U)^n$ Y, $(I-U)^n$ Z, $(I-U)^n$T]dU

$$=LF_{10}{}^{4}\left[\begin{matrix} a_{1},a_{1},a_{2},a_{2},b_{1},b_{1},b_{1},b_{1},c_{1},c_{2},c_{1},c_{3;},\Delta(n,b),\Delta(\frac{n}{2},\frac{c+b}{2}), \\ ,\Delta(\frac{n}{2},\frac{1+c+b}{2}),\Delta(n,c+b),\Delta\left(\frac{n}{2},c+b+a\right),\Delta(\frac{n}{2},\frac{1+c+b-a}{2}); X; Y; Z; T\end{matrix}\right]$$ ……………

………………………………………………………(2.1.10)

Where

$F_{10}{}^{4}\,[a_{1},a_{1},a_{2},a_{2},b_{1},b_{1},b_{1},b_{1},c_{1},c_{2},c_{1},c_{3;},X,Y,Z,T]$

$=\sum_{p,q,r,s=0}^{\infty}\frac{(a_{1})_{p+q}(a_{2})_{r+s}(b_{1})_{p+q+r+s}}{p!s!q!r!(c_{1})_{p+r}\,(c_{2})_{q}\,(c_{3})_{r}}\; X^{p}\; Y^{q}\; Z^{r}\; T^{s}$

Then probability density function (p.d.f) is given by :

$$F(U) = \frac{U^{a-\frac{m+1}{2}}\,(I-U)^{b-\frac{m+1}{2}}\,F_{1}(X_{1})F_{10}{}^{4}\,[X_{2}]}{LF_{10}{}^{4}\,[X_{12}]}$$

=0 elsewhere

Where

$$X_{12}=\left[\begin{matrix} a_{1},a_{1},a_{2},a_{2},b_{1},b_{1},b_{1},b_{1},c_{1},c_{2},c_{1},c_{3;},\Delta(n,b),\Delta(\frac{n}{2},\frac{c+b}{2}), \\ \Delta(\frac{n}{2},\frac{1+c+b}{2})\,\Delta(n,c+b),\Delta\left(\frac{n}{2},c+b+a\right),\Delta(\frac{n}{2},\frac{1+c+b-a}{2}); X; Y; Z; T\end{matrix}\right]$$

[11] $\int_{0}^{I} U^{a-\frac{m+1}{2}}\,(I-U)^{b-\frac{m+1}{2}}\,F_{1}$ (a,1-a;c;$\frac{U}{2}$).

$F_{11}{}^{4}\,[(I-U)^{n}$ X,$(I-U)^{n}$ Y, $(I-U)^{n}$ Z, $(I-U)^{n}$T]dU

$$=LF_{11}{}^{4}\left[\begin{matrix} a_{1},a_{1},a_{2},a_{3},b_{1},b_{1},b_{1},b_{1},c_{1},c_{2},c_{1},c_{3;},\Delta(n,b),\Delta(\frac{n}{2},\frac{c+b}{2}), \\ ,\Delta(\frac{n}{2},\frac{1+c+b}{2}),\Delta(n,c+b),\Delta\left(\frac{n}{2},c+b+a\right),\Delta(\frac{n}{2},\frac{1+c+b-a}{2}); X; Y; Z; T\end{matrix}\right]$$ ……………

………………………………………………………(2.1.11)

Where

$F_{11}{}^{4}\,[a_{1},a_{1},a_{2},a_{3},b_{1},b_{1},b_{1},b_{1},c_{1},c_{2},c_{1},c_{3;},X,Y,Z,T]$

$=\sum_{p,q,r,s=0}^{\infty}\frac{(a_{1})_{p+q}(a_{2})_{r}(a_{3})_{s}\,(b_{1})_{p+q+r+s}}{p!s!q!r!(c_{1})_{p+r}\,(c_{2})_{q}\,(c_{3})_{r}}\; X^{p}\; Y^{q}\; Z^{r}\; T^{s}$

Then probability density function (p.d.f) of (2.1.11) is given by :

F(U) = $\frac{U^{a-\frac{m+1}{2}}\ (I-U)^{b-\frac{m+1}{2}}\ F_1(X_1)F_{11}{}^4\,[X_2]}{LF_{11}{}^4\,[X_{13}]}$

=0 elsewhere

Where

$$X_{13} =$$

$$\begin{bmatrix} a_{1},, a_{1},, a_2\ , a_{3}, b_1, b_{1,}, b_{1,} b_{1}, c_{1}, c_{2}, c_{1}, c_{3;}, \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ \Delta(\frac{n}{2}, \frac{1+c+b}{2})\ \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{bmatrix}$$

[12] $\int_0^I U^{a-\frac{m+1}{2}}\ (I-U)^{b-\frac{m+1}{2}}\ F_1$ (a,1-a;c;$\frac{U}{2}$).

$F_{12}{}^4$ [$(I-U)^n$ X,$(I-U)^n$ Y, $(I-U)^n$ Z, $(I-U)^n$T]dU

$$=LF_{12}{}^4 \begin{bmatrix} a_{1},, a_{1},, a_{2},, a_{3}, b_{1}, b_{1},, b_{1}, b_{1}, c_{1}, c_{2}, c_{3}, c_{3;}, \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ , \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{bmatrix} \ldots\ldots\ldots\ldots\ldots$$

………………..……………………………………(2.1.12)

Where

$F_{12}{}^4\ [a_{1},, a_{1},, a_{2},, a_{3}, b_{1}, b_{1},, b_{1}, b_{1}, c_{1}, c_{2}, c_{3}, c_{3;}, X, Y, Z, T]$

$$=\sum_{p,q,r,s=0}^{\infty} \frac{(a_{1})_{p+q}(a_{2})_r\,(a_{3})_s(b_{1})_{p+q+r+s}}{p!s!q!r!(c_{1})_p\ (C_{2})_q\ (C_{3})_{r+s}}\ X^p\ Y^q\ Z^r\ T^s$$

Then probability density function (p.d.f) of (2.1.12) is given by :

F(U) = $\frac{U^{a-\frac{m+1}{2}}\ (I-U)^{b-\frac{m+1}{2}}\ F_1(X_1)F_{12}{}^4\,[X_2]}{LF_{12}{}^4\,[X_{14}]}$

=0 elsewhere

Where

$X_{14} =$

$$\left[\begin{array}{c} a_{1,}, a_{1,}, a_{2}, a_{3,} b_{1,} b_{1,}, b_{1,} b_{1,} c_{1,} c_{2,} c_{3;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ \Delta\left(\frac{n}{2}, \frac{1+c+b}{2}\right), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{array}\right]$$

[13] $\int_0^I U^{a-\frac{m+1}{2}} \ (I-U)^{b-\frac{m+1}{2}} \ F_1$ (a,1-a;c;$\frac{U}{2}$).

${F_{13}}^4$ [$(I-U)^n$ X,$(I-U)^n$ Y, $(I-U)^n$ Z, $(I-U)^n$T]dU

$= L{F_{13}}^4 \left[\begin{array}{c} a_{1,}, a_{2,}, a_{3,}, a_{4,} b_{1,} b_{1,}, b_{1,} b_{1,} c_{1,} c_{1,} c_{2,} c_{3;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ , \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{array}\right]$……………

……..………………………………………………(2.1.13)

Where

${F_{13}}^4$ $[a_{1,}, a_{2,}, a_{3,}, a_{4,} b_{1,} b_{1,}, b_{1,} b_{1,} c_{1,} c_{1,} c_{2,} c_{3;,} X, Y, Z, T]$

$=\sum_{p,q,r,s=0}^{\infty} \frac{(a_{1,})_p (a_{2,})_r (a_{3,})_r (a_{4,})_s (b_{1,})_{p+q+r+s}}{p!s!q!r!(c_{1,})_{p+q} \ (c_{2,})_r \ (C_{3,})_s} \quad X^p \ \ Y^q \ Z^r \ T^s$

Then probability density function (p.d.f) of (2.1.13) is given by :

F(U) $=\frac{U^{a-\frac{m+1}{2}} \ (I-U)^{b-\frac{m+1}{2}} \ F_1(X_1){F_{13}}^4 \ [X_2]}{L{F_{13}}^4 \ [X_{15}]}$

=0 elsewhere

Where

$$X_{15} =$$

$$\left[\begin{array}{c} a_{1,}, a_{2,}, a_{3}, a_{4,} b_{1,} b_{1,}, b_{1,} b_{1,} c_{1,} c_{1,} c_{2,} c_{3;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ \Delta\left(\frac{n}{2}, \frac{1+c+b}{2}\right), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{array}\right]$$

[14] $\int_0^I U^{a-\frac{m+1}{2}} \ (I-U)^{b-\frac{m+1}{2}} \ F_1$ (a,1-a;c;$\frac{U}{2}$).

${F_{14}}^4$ [$(I-U)^n$ X,$(I-U)^n$ Y, $(I-U)^n$ Z, $(I-U)^n$T]dU

$$=LF_{14}{}^{4}\left[\begin{array}{c} a_{1}, a_{1}, a_{1}, a_{2}, b_{1}, b_{1}, b_{1}, b_{2}, c_{1}, c_{2}, c_{3}, c_{1};, \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ ,\Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); \mathrm{X}; \mathrm{Y}; \mathrm{Z}; \mathrm{T} \end{array}\right] \ldots\ldots\ldots\ldots\ldots$$

……………………………..………………………(2.1.14)

Where

$F_{14}{}^{4}\,[a_{1}, a_{1}, a_{1}, a_{2}, b_{1}, b_{1}, b_{1}, b_{2}, c_{1}, c_{2}, c_{3}, c_{1};, X, Y, Z, T]$

$$=\sum_{p,q,r,s=0}^{\infty} \frac{(a_{1})_{p+q+r}(a_{2})_{s}(b_{1})_{p+q+r}(b_{2})_{s}}{p!s!q!r!(c_{1})_{p+s}\,(C_{2})_{q}\,(C_{3})_{r}} \quad X^{p}\;\; Y^{q}\; Z^{r}\; T^{s}$$

Then probability density function (p.d.f) of (2.1.14) is given by :

$$\mathrm{F(U)} = \frac{U^{a-\frac{m+1}{2}}\;\; (I-U)^{b-\frac{m+1}{2}}\; F_{1}(X_{1})F_{14}{}^{4}\,[X_{2}]}{LF_{14}{}^{4}\,[X_{16}]}$$

=0 elsewhere

Where

$X_{16} =$

$$\left[\begin{array}{c} a_{1}, a_{1}, a_{1}, a_{2}, b_{1}, b_{1}, b_{1}, b_{2}, c_{1}, c_{2}, c_{3}, c_{1};, \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ \Delta\left(\frac{n}{2}, \frac{1+c+b}{2}\right), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); \mathrm{X}; \mathrm{Y}; \mathrm{Z}; \mathrm{T} \end{array}\right]$$

[15] $\int_{0}^{I} U^{a-\frac{m+1}{2}}\;\; (I-U)^{b-\frac{m+1}{2}}\;\; F_{1}$ (a,1-a;c;$\frac{U}{2}$).

$F_{15}{}^{4}$ [$(I-U)^{n}$ X,$(I-U)^{n}$ Y, $(I-U)^{n}$ Z, $(I-U)^{n}$T]dU

$$=LF_{15}{}^{4}\left[\begin{array}{c} a_{1}, a_{1}, a_{1}, a_{2}, b_{1}, b_{1}, b_{1}, b_{2}, c_{1}, c_{2}, c_{1}, c_{3};, \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ ,\Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); \mathrm{X}; \mathrm{Y}; \mathrm{Z}; \mathrm{T} \end{array}\right] \ldots\ldots\ldots\ldots\ldots$$

…………..…………………………………………(2.1.15)

Where

$F_{15}{}^{4}\,[a_{1}, a_{1}, a_{1}, a_{2}, b_{1}, b_{1}, b_{1}, b_{2}, c_{1}, c_{2}, c_{1}, c_{3};, X, Y, Z, T]$

$$=\sum_{p,q,r,s=0}^{\infty} \frac{(a_{1})_{p+q+r}(a_{2})_{s}(b_{1})_{p+q+r}(b_{2})_{s}}{p!s!q!r!(c_{1})_{p+r}\,(C_{2})_{q}\,(C_{3})_{s}} \quad X^{p}\;\; Y^{q}\; Z^{r}\; T^{s}$$

Then probability density function (p.d.f) of (2.1.15) is given by :

$$F(U) = \frac{U^{a-\frac{m+1}{2}} \ (I-U)^{b-\frac{m+1}{2}} \ F_1(X_1) {F_{15}}^4 \ [X_2]}{L {F_{15}}^4 \ [X_{17}]}$$

=0 elsewhere

Where

$X_{17} =$

$$\left[\begin{matrix} a_{1\,,}, a_{1\,,}, a_1, a_{2\,,} b_{1\,,} b_{1,}, b_{1,} b_{2\,,} c_{1\,,} c_{2\,,} c_{1\,,} c_{3;\,,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ \Delta\left(\frac{n}{2}, \frac{1+c+b}{2}\right), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{matrix}\right]$$

[16] $\int_0^I U^{a-\frac{m+1}{2}} \ (I-U)^{b-\frac{m+1}{2}} \ F_1$ (a,1-a;c;$\frac{U}{2}$).

${F_{16}}^4$ [$(I-U)^n$ X,$(I-U)^n$ Y, $(I-U)^n$ Z, $(I-U)^n$T]dU

$$= L{F_{16}}^4 \left[\begin{matrix} a_{1\,,}, a_{1\,,}, a_{1\,,}, a_{2\,,} b_{1\,,} b_{1\,,}, b_{1\,,} b_{2\,,} c_{1\,,} c_{2\,,} c_{3,} c_{3;\,,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ , \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{matrix}\right] \ldots\ldots\ldots\ldots\ldots$$

………………………………………………………(2.1.16)

Where

${F_{16}}^4$ $[a_{1\,,}, a_{1\,,}, a_{1\,,}, a_{2\,,} b_{1\,,} b_{1\,,}, b_{2\,,} b_{1\,,} c_{1\,,} c_{2\,,} c_{3\,,} c_{3;\,,} X, Y, Z, T]$

$$= \sum_{p,q,r,s=0}^{\infty} \frac{(a_{1\,,})_{p+q+r} (a_{2\,,})_s (b_{1\,,})_{p+q+r} (b_{2\,,})_r}{p!s!q!r!(c_{1,})_p \ (c_{2\,,})_q \ (C_{3\,,})_{r+s}} \ X^p \ Y^q \ Z^r \ T^s$$

Then probability density function (p.d.f) of (2.1.16) is given by :

$$F(U) = \frac{U^{a-\frac{m+1}{2}} \ (I-U)^{b-\frac{m+1}{2}} \ F_1(X_1) {F_{16}}^4 \ [X_2]}{L {F_{16}}^4 \ [X_{18}]}$$

=0 elsewhere

Where

$X_{18} =$

$$\begin{bmatrix} a_{1,}, a_{1,}, a_1, a_{2,} b_{1,} b_{1,}, b_{1,} b_{1,} c_{1,} c_{2,} c_{3,} c_{3;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ \Delta\left(\frac{n}{2}, \frac{1+c+b}{2}\right), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{bmatrix}$$

[17] $\int_0^I U^{a-\frac{m+1}{2}}\ (I-U)^{b-\frac{m+1}{2}}\ F_1$ (a,1-a;c;$\frac{U}{2}$).

${F_{17}}^4$ [$(I-U)^n$ X,$(I-U)^n$ Y, $(I-U)^n$ Z, $(I-U)^n$T]dU

$$=L{F_{17}}^4 \begin{bmatrix} a_{1,}, a_{1,}, a_{1,}, a_{2,} b_{1,} b_{1,}, b_{2,} b_{2,} c_{1,} c_{2,} c_{1,} c_{3;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ , \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{bmatrix} \ldots\ldots\ldots\ldots\ldots$$

……………………………………………………(2.1.17)

Where

${F_{17}}^4$ $[a_{1,}, a_{1,}, a_{1,}, a_{2,} b_{1,} b_{1,}, b_{2,} b_{2,} c_{1,} c_{2,} c_{1,} c_{3;,} X, Y, Z, T]$

$$=\sum_{p,q,r,s=0}^{\infty} \frac{(a_{1,})_{p+q+r}(a_{2,})_s(b_{1,})_{p+q+}(b_{2,})_{r+s}}{p!s!q!r!(c_{1,})_{p+q}\ (C_{2,})_q\ (C_{3,})_s}\ \ X^p\ \ Y^q\ Z^r\ T^s$$

Then probability density function (p.d.f) of (2.1.17) is given by :

$$F(U) = \frac{U^{a-\frac{m+1}{2}}\ (I-U)^{b-\frac{m+1}{2}}\ F_1(X_1){F_{17}}^4\ [X_2]}{L{F_{17}}^4\ [X_{19}]}$$

=0 elsewhere

Where

$X_{19} =$

$$\begin{bmatrix} a_{1,}, a_{1,}, a_1, a_{2,} b_{1,} b_{1,}, b_{2,} b_{2,} c_{1,} c_{2,} c_{1,} c_{3;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ \Delta\left(\frac{n}{2}, \frac{1+c+b}{2}\right), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{bmatrix}$$

[18] $\int_0^I U^{a-\frac{m+1}{2}}\ (I-U)^{b-\frac{m+1}{2}}\ F_1$ (a,1-a;c;$\frac{U}{2}$).

${F_{18}}^4$ [$(I-U)^n$ X,$(I-U)^n$ Y, $(I-U)^n$ Z, $(I-U)^n$T]dU

$=LF_{18}{}^{4}\begin{bmatrix} a_{1},a_{1},a_{1},a_{2},b_{1},b_{1},b_{2},b_{2},c_{1},c_{2},c_{3},c_{1};,\Delta(n,b),\Delta(\frac{n}{2},\frac{c+b}{2}), \\ ,\Delta(\frac{n}{2},\frac{1+c+b}{2}),\Delta(n,c+b),\Delta\left(\frac{n}{2},c+b+a\right),\Delta(\frac{n}{2},\frac{1+c+b-a}{2}); X; Y; Z; T \end{bmatrix}$……………
…………………………………………………(2.1.18)

Where

$F_{18}{}^{4}\,[a_{1},a_{1},a_{1},a_{2},b_{1},b_{1},b_{2},b_{2},c_{1},c_{2},c_{3},c_{1};,X,Y,Z,T]$

$=\sum_{p,q,r,s=0}^{\infty}\frac{(a_{1})_{p+q+r}(a_{2})_{s}(b_{1})_{p+q}(b_{2})_{r+s}}{p!s!q!r!(c_{1})_{p+s}(C_{2})_{q}(C_{3})_{r}}\;X^{p}\;Y^{q}\;Z^{r}\;T^{s}$

Then probability density function (p.d.f) of (2.1.18) is given by :

$F(U)=\frac{U^{a-\frac{m+1}{2}}\;(I-U)^{b-\frac{m+1}{2}}\;F_{1}(X_{1})F_{18}{}^{4}\,[X_{2}]}{LF_{18}{}^{4}\,[X_{20}]}$

=0 elsewhere

Where

$X_{20}=$

$$\begin{bmatrix} a_{1},a_{1},a_{1},a_{2},b_{1},b_{1},b_{2},b_{2},c_{1},c_{2},c_{3},c_{1};,\Delta(n,b),\Delta(\frac{n}{2},\frac{c+b}{2}), \\ \Delta\left(\frac{n}{2},\frac{1+c+b}{2}\right),\Delta(n,c+b),\Delta\left(\frac{n}{2},c+b+a\right),\Delta(\frac{n}{2},\frac{1+c+b-a}{2}); X; Y; Z; T \end{bmatrix}$$

[19] $\int_{0}^{I}U^{a-\frac{m+1}{2}}\;(I-U)^{b-\frac{m+1}{2}}\;F_{1}$ (a,1-a;c;$\frac{U}{2}$).

$F_{19}{}^{4}\,[(I-U)^{n}$ X,$(I-U)^{n}$ Y, $(I-U)^{n}$ Z, $(I-U)^{n}$T]dU

$=LF_{19}{}^{4}\begin{bmatrix} a_{1},a_{1},a_{1},a_{2},b_{1},b_{1},b_{2},b_{2},c_{1},c_{2},c_{3},c_{3};,\Delta(n,b),\Delta(\frac{n}{2},\frac{c+b}{2}), \\ ,\Delta(\frac{n}{2},\frac{1+c+b}{2}),\Delta(n,c+b),\Delta\left(\frac{n}{2},c+b+a\right),\Delta(\frac{n}{2},\frac{1+c+b-a}{2}); X; Y; Z; T \end{bmatrix}$……………
………..……………………………………………(2.1.19)

Where

$F_{19}{}^{4}\,[a_{1},a_{1},a_{1},a_{2},b_{1},b_{1},b_{2},b_{2},c_{1},c_{2},c_{3},c_{3};,X,Y,Z,T]$

$=\sum_{p,q,r,s=0}^{\infty}\frac{(a_{1})_{p+q+r}(a_{2})_{s}(b_{1})_{p+q}(b_{2})_{r+s}}{p!s!q!r!(c_{1})_{p}(C_{2})_{q}(C_{3})_{r+s}}\;X^{p}\;Y^{q}\;Z^{r}\;T^{s}$

Then probability density function (p.d.f) of (2.1.19) is given by :

$$F(U) = \frac{U^{a-\frac{m+1}{2}} \; (I-U)^{b-\frac{m+1}{2}} \; F_1(X_1) {F_{19}}^4 [X_2]}{L {F_{19}}^4 [X_{21}]}$$

=0 elsewhere

Where

$X_{21} =$

$$\begin{bmatrix} a_{1,}, a_{1,}, a_1, a_{2,} b_{1,} b_{1,}, b_{2,} b_{2,} c_{1,} c_{2,} c_{3,} c_{3;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ \Delta\left(\frac{n}{2}, \frac{1+c+b}{2}\right), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); \mathrm{X}; \mathrm{Y}; \mathrm{Z}; \mathrm{T} \end{bmatrix}$$

$$= L {F_{20}}^4 \begin{bmatrix} a_{1,}, a_{1,}, a_{1,}, a_{2,} b_{1,} b_{1,}, b_{3,} b_{3,} c_{1,} c_{2,} c_{3,} c_{1;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ , \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); \mathrm{X}; \mathrm{Y}; \mathrm{Z}; \mathrm{T} \end{bmatrix}$$ ……………

…………………………………………………………(1.2.20)

Where

${F_{20}}^4 \, [a_{1,}, a_{1,}, a_{1,}, a_{2,} b_{1,} b_{1,}, b_{3,} b_{3,} c_{1,} c_{2,} c_{3,} c_{1;,} X, Y, Z, T]$

$$= \sum_{p,q,r,s=0}^{\infty} \frac{(a_{1,})_{p+q+r} (a_{2,})_s (b_{1,})_{p+q} (b_{2,})_{r+s} (b_{3,})_s}{p!s!q!r!(c_{1,})_p (C_{2,})_q (C_{3,})_{r+s}} \; X^p \; Y^q \; Z^r \; T^s$$

Then probability density function (p.d.f) of (1.2.20) is given by :

$$F(U) = \frac{U^{a-\frac{m+1}{2}} \; (I-U)^{b-\frac{m+1}{2}} \; F_1(X_1) {F_{20}}^4 [X_2]}{L {F_{20}}^4 [X_{22}]}$$

=0 elsewhere

Where

$X_{22} =$

$$\begin{bmatrix} a_{1,}, a_{1,}, a_1, a_{2,} b_{1,} b_{1,}, b_{3,} b_{3,} c_{1,} c_{2,} c_{3,} c_{1;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ \Delta\left(\frac{n}{2}, \frac{1+c+b}{2}\right), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); \mathrm{X}; \mathrm{Y}; \mathrm{Z}; \mathrm{T} \end{bmatrix}$$

[21] $\int_0^I U^{a-\frac{m+1}{2}}\ (I-U)^{b-\frac{m+1}{2}}\ F_1$ (a,1-a;c;$\frac{U}{2}$).

${F_{21}}^4$ [$(I-U)^n$ X,$(I-U)^n$ Y, $(I-U)^n$ Z, $(I-U)^n$T]dU

$$=L{F_{21}}^4\begin{bmatrix} a_1, a_1, a_1, a_2, b_1, b_1, b_2, b_3, c_1, c_2, c_3, c_{3;}, \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ ,\Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{bmatrix}$$ ……………

……………………………………………………………(1.2.21)

Where

${F_{21}}^4$ $[a_1, a_1, a_1, a_2, b_1, b_1, b_2, b_3, c_1, c_2, c_3, c_{3;}, X, Y, Z, T]$

$$=\sum_{p,q,r,s=0}^{\infty} \frac{(a_1)_{p+q+r}(a_2)_s(b_1)_{p+q}(b_2)_{r}(b_3)_s}{p!s!q!r!(c_1)_p(C_2)_q(C_3)_{r+s}}\ X^p\ Y^q\ Z^r\ T^s$$

Then probability density function (p.d.f) of (1.2.21) is given by :

$$F(U) = \frac{U^{a-\frac{m+1}{2}}\ (I-U)^{b-\frac{m+1}{2}}\ F_1(X_1){F_{21}}^4\ [X_2]}{L{F_{21}}^4\ [X_{23}]}$$

=0 elsewhere

Where

$$X_{23} =$$

$$\begin{bmatrix} a_1, a_1, a_1, a_2, b_1, b_1, b_2, b_3, c_1, c_2, c_3, c_{3;}, \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ \Delta\left(\frac{n}{2}, \frac{1+c+b}{2}\right), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{bmatrix}$$

[22] $\int_0^I U^{a-\frac{m+1}{2}}\ (I-U)^{b-\frac{m+1}{2}}\ F_1$ (a,1-a;c;$\frac{U}{2}$).

${F_{22}}^4$ [$(I-U)^n$ X,$(I-U)^n$ Y, $(I-U)^n$ Z, $(I-U)^n$T]dU

$$=L{F_{22}}^4\begin{bmatrix} a_1, a_1, a_1, a_2, b_1, b_1, b_2, b_3, c_1, c_1, c_2, c_{3;}, \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ ,\Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{bmatrix}$$ ……………

……………………………………………………………(1.2.22)

Where

$F_{22}{}^{4}\ [a_{1,}, a_{1,}, a_{1,}, a_{2,} b_{1,} b_{2,}, b_{3,} b_{1,} c_{1,} c_{1,} c_{2,} c_{3;,} X, Y, Z, T]$

$=\sum_{p,q,r,s=0}^{\infty} \frac{(a_{1,})_{p+q+r}(a_{2,})_{s}(b_{1,})_{p+s}(b_{2,})_{q}(b_{3,})_{r}}{p!s!q!r!(c_{1,})_{p+q}(c_{2,})_{r}(C_{3,})_{s}} \quad X^{p}\ \ Y^{q}\ Z^{r}\ T^{s}$

Then probability density function (p.d.f) of (1.2.22) is given by :

$\text{F(U)} = \frac{U^{a-\frac{m+1}{2}}\ (I-U)^{b-\frac{m+1}{2}}\ F_1(X_1) F_{22}{}^{4}\ [X_2]}{L F_{22}{}^{4}\ [X_{24}]}$

=0 elsewhere

Where

$$X_{24} =$$

$$\left[\begin{array}{c} a_{1,}, a_{1,}, a_{1}, a_{2,} b_{1,} b_{2,}, b_{3,} b_{1,} c_{1,} c_{1,} c_{2,} c_{3;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ \Delta\left(\frac{n}{2}, \frac{1+c+b}{2}\right), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); \text{X; Y; Z; T} \end{array}\right]$$

[23] $\int_0^I U^{a-\frac{m+1}{2}}\ (I-U)^{b-\frac{m+1}{2}}\ F_1$ (a,1-a;c;$\frac{U}{2}$).

$F_{23}{}^{4}\ [(I-U)^{n}\ \text{X}, (I-U)^{n}\ \text{Y}, (I-U)^{n}\ \text{Z}, (I-U)^{n}\text{T}]\text{dU}$

$$=L F_{23}{}^{4} \left[\begin{array}{c} a_{1,}, a_{1,}, a_{1,}, a_{2,} b_{1,} b_{2,}, b_{3,} b_{1,} c_{1,} c_{2,} c_{2,} c_{3;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ , \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); \text{X; Y; Z; T} \end{array}\right] \ldots\ldots\ldots\ldots\ldots$$

………………………………..…………………(1.2.23)

Where

$F_{23}{}^{4}\ [a_{1,}, a_{1,}, a_{1,}, a_{2,} b_{1,} b_{2,}, b_{3,} b_{1,} c_{1,} c_{2,} c_{2,} c_{3;,} X, Y, Z, T]$

$=\sum_{p,q,r,s=0}^{\infty} \frac{(a_{1,})_{p+q+r}(a_{2,})_{s}(b_{1,})_{p+s}(b_{2,})_{q}(b_{3,})_{r}}{p!s!q!r!(c_{1,})_{p}(c_{2,})_{q+r}(C_{3,})_{s}} \quad X^{p}\ \ Y^{q}\ Z^{r}\ T^{s}$

Then probability density function (p.d.f) of (1.2.23) is given by :

$\text{F(U)} = \frac{U^{a-\frac{m+1}{2}}\ (I-U)^{b-\frac{m+1}{2}}\ F_1(X_1) F_{23}{}^{4}\ [X_2]}{L F_{23}{}^{4}\ [X_{25}]}$

=0 elsewhere

Where

$$X_{25} =$$

$$\begin{bmatrix} a_{1\,,}, a_{1\,,}, a_{1}\,, a_{2\,,} b_{1\,,} b_{2,}, b_{3,} b_{1\,,} c_{1\,,} c_{2\,,} c_{2\,,} c_{3;\,,} \Delta(n,b), \Delta(\frac{n}{2}\,, \frac{c+b}{2})\,, \\ \Delta\left(\frac{n}{2}\,, \frac{1+c+b}{2}\right), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}\,, \frac{1+c+b-a}{2}); X; Y; Z; T \end{bmatrix}$$

[24] $\int_0^I U^{a-\frac{m+1}{2}}\ (I-U)^{b-\frac{m+1}{2}}\ F_1$ (a,1-a;c;$\frac{U}{2}$).

${F_{24}}^4$ $[(I-U)^n$ X,$(I-U)^n$ Y, $(I-U)^n$ Z, $(I-U)^n$T]dU

$$=L{F_{24}}^4 \begin{bmatrix} a_{1\,,}, a_{1\,,}, a_{1\,,}, a_{2\,,} b_{1\,,} b_{2\,,}, b_{3\,,} b_{1\,,} c_{1\,,} c_{2,} c_{2,} c_{3;\,,} \Delta(n,b), \Delta(\frac{n}{2}\,, \frac{c+b}{2})\,, \\ , \Delta(\frac{n}{2}\,, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}\,, \frac{1+c+b-a}{2}); X; Y; Z; T \end{bmatrix} \ldots\ldots\ldots\ldots\ldots$$

……………………………………………………..(1.2.24)

Where

${F_{24}}^4$ $[a_{1\,,}, a_{1\,,}, a_{1\,,}, a_{2\,,} b_{1\,,} b_{2\,,}, b_{3\,,} b_{1\,,} c_{1\,,} c_{2\,,} c_{2\,,} c_{3;\,,} X, Y, Z, T]$

$$=\sum_{p,q,r,s=0}^{\infty} \frac{(a_{1\,,})_{p+q+r} (a_{2\,,})_s (b_{1\,,})_{p+s} (b_{2\,,})_q\, (b_{3\,,})_r}{p!s!q!r!(c_{1,})_p\, (c_{2\,,})_{q+r}\, (C_{3\,,})_s}\ \ X^p\ \ Y^q\ Z^r\ T^s$$

Then probability density function (p.d.f) of (1.2.24) is given by :

$$\text{F(U)} = \frac{U^{a-\frac{m+1}{2}}\ (I-U)^{b-\frac{m+1}{2}}\ F_1(X_1) {F_{24}}^4\,[X_2]}{L{F_{24}}^4\,[X_{26}]}$$

=0 elsewhere

Where

$$X_{26} =$$

$$\begin{bmatrix} a_{1\,,}, a_{1\,,}, a_{1}\,, a_{2\,,} b_{1\,,} b_{2,}, b_{3,} b_{1\,,} c_{1\,,} c_{2\,,} c_{2\,,} c_{3;\,,} \Delta(n,b), \Delta(\frac{n}{2}\,, \frac{c+b}{2})\,, \\ \Delta\left(\frac{n}{2}\,, \frac{1+c+b}{2}\right), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}\,, \frac{1+c+b-a}{2}); X; Y; Z; T \end{bmatrix}$$

[25] $\int_0^I U^{a-\frac{m+1}{2}}\ (I-U)^{b-\frac{m+1}{2}}\ F_1$ (a,1-a;c;$\frac{U}{2}$).

${F_{25}}^4$ $[(I-U)^n$ X,$(I-U)^n$ Y, $(I-U)^n$ Z, $(I-U)^n$T]dU

$$=LF_{25}{}^{4}\begin{bmatrix} a_{1,}, a_{1,}, a_{1,}, a_{2,} b_{1,} b_{2,}, b_{3,} b_{1,} c_{1,} c_{2,} c_{3,} c_{1;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ , \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{bmatrix} \ldots\ldots\ldots\ldots\ldots$$

…………………………………………………………..(1.2.25)

Where

$F_{25}{}^{4}\,[a_{1,}, a_{1,}, a_{2,}, a_{1,} b_{1,} b_{2,}, b_{3,} b_{1,} c_{1,} c_{2,} c_{3,} c_{1;,} X, Y, Z, T]$

$$=\sum_{p,q,r,s=0}^{\infty} \frac{(a_{1,})_{p+q+r}(a_{2,})_{s}(b_{1,})_{p+s}(b_{2,})_{q}(b_{3,})_{r}}{p!s!q!r!(c_{1,})_{p+s}(C_{2,})_{q}(C_{3,})_{r}} \; X^{p} \; Y^{q} \, Z^{r} \, T^{s}$$

Then probability density function (p.d.f) of (1.2.25) is given by :

$$F(U) = \frac{U^{a-\frac{m+1}{2}} \; (I-U)^{b-\frac{m+1}{2}} \; F_{1}(X_{1}) F_{25}{}^{4}\,[X_{2}]}{LF_{25}{}^{4}\,[X_{27}]}$$

=0 elsewhere

Where

$X_{27} =$

$$\begin{bmatrix} a_{1,}, a_{1,}, a_{1}, a_{2,} b_{1,} b_{2,}, b_{3,} b_{1,} c_{1,} c_{2,} c_{3,} c_{1;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ \Delta\left(\frac{n}{2}, \frac{1+c+b}{2}\right), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{bmatrix}$$

[26] $\int_{0}^{I} U^{a-\frac{m+1}{2}} \; (I-U)^{b-\frac{m+1}{2}} \; F_{1}$ (a,1-a;c;$\frac{U}{2}$).

$F_{26}{}^{4}\,[(I-U)^{n}$ X,$(I-U)^{n}$ Y, $(I-U)^{n}$ Z, $(I-U)^{n}$T]dU

$$=LF_{26}{}^{4}\begin{bmatrix} a_{1,}, a_{1,}, a_{1,}, a_{2,} b_{1,} b_{2,}, b_{3,} b_{4,} c_{1,} c_{2,} c_{3,} c_{1;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ , \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{bmatrix} \ldots\ldots\ldots\ldots\ldots$$

……………………………………………………….(1.2.26)

Where

$F_{26}{}^{4}\,[a_{1,}, a_{1,}, a_{1,}, a_{2,} b_{1,} b_{2,}, b_{3,} b_{4,} c_{1,} c_{2,} c_{3,} c_{1;,} X, Y, Z, T]$

$$=\sum_{p,q,r,s=0}^{\infty} \frac{(a_{1,})_{p+q+r}(a_{2,})_{s}(b_{1,})_{p}(b_{2,})_{q}(b_{3,})_{r}(b_{4,})_{s}}{p!s!q!r!(c_{1,})_{p+s}(C_{2,})_{q}(C_{3,})_{r}} \; X^{p} \; Y^{q} \, Z^{r} \, T^{s}$$

Then probability density function (p.d.f) of (1.2.26) is given by :

F(U) =$\frac{U^{a-\frac{m+1}{2}}\ (I-U)^{b-\frac{m+1}{2}}\ F_1(X_1)F_{26}{}^4\,[X_2]}{LF_{26}{}^4\,[X_{28}]}$

=0 elsewhere

Where

$X_{28} =$

$$\left[\begin{array}{c} a_{1\,,}, a_{1\,,}, a_1\,, a_{2\,,}b_{1\,,}b_{2,}, b_{3,}b_{4\,,}c_{1\,,}c_{2\,,}c_{3\,,}c_{1;\,,}\Delta(n,b), \Delta(\frac{n}{2}\,, \frac{c+b}{2})\,, \\ \Delta\left(\frac{n}{2}\,, \frac{1+c+b}{2}\right), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}\,, \frac{1+c+b-a}{2}); X; Y; Z; T \end{array}\right]$$

[27] $\int_0^I U^{a-\frac{m+1}{2}}\ (I-U)^{b-\frac{m+1}{2}}\ F_1$ (a,1-a;c;$\frac{U}{2}$).

$F_{27}{}^4$ [$(I-U)^n$ X,$(I-U)^n$ Y, $(I-U)^n$ Z, $(I-U)^n$T]dU

$$=LF_{27}{}^4\left[\begin{array}{c} a_{1\,,}, a_{1\,,}, a_{2\,,}, a_{2\,,}b_{1\,,}b_{2\,,}, b_{1\,,}b_{2\,,}c_{1\,,}c_{2,}c_{2,}c_{3;\,,}\Delta(n,b), \Delta(\frac{n}{2}\,, \frac{c+b}{2})\,, \\ , \Delta(\frac{n}{2}\,, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}\,, \frac{1+c+b-a}{2}); X; Y; Z; T \end{array}\right]\ldots\ldots\ldots\ldots\ldots$$

……………..…………………………………..(1.2.27)

Where

$F_{27}{}^4$ [$a_{1\,,}, a_{1\,,}, a_{2\,,}, a_{2\,,}b_{1\,,}b_{2\,,}, b_{1\,,}b_{2\,,}c_{1\,,}c_{2\,,}c_{1\,,}c_{3;\,,}X, Y, Z, T$]

$=\sum_{p,q,r,s=0}^{\infty} \frac{(a_{1\,,})_{p+q}(a_{2\,,})_{r+s}(b_{1\,,})_{p+s}(b_{2\,,})_{q+s}}{p!s!q!r!(c_{1,})_{p+r}\,(c_{2\,,})_q\,(C_{3\,,})_s}\ \ X^p\ \ Y^q\ Z^r\ T^s$

Then probability density function (p.d.f) of (1.2.27) is given by :

F(U) =$\frac{U^{a-\frac{m+1}{2}}\ (I-U)^{b-\frac{m+1}{2}}\ F_1(X_1)F_{27}{}^4\,[X_2]}{LF_{27}{}^4\,[X_{29}]}$

=0 elsewhere

Where

$X_{29} =$

$$\left[\begin{array}{c} a_{1,}, a_{1,}, a_{2}, a_{2,} b_{1,} b_{2,}, b_{1,} b_{2,} c_{1,} c_{2,} c_{2,} c_{3;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ \Delta\left(\frac{n}{2}, \frac{1+c+b}{2}\right), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{array}\right]$$

[28] $\int_0^I U^{a-\frac{m+1}{2}} \ (I-U)^{b-\frac{m+1}{2}} \ F_1$ (a,1-a;c;$\frac{U}{2}$).

${F_{28}}^4$ [$(I-U)^n$ X,$(I-U)^n$ Y, $(I-U)^n$ Z, $(I-U)^n$T]dU

$$=L{F_{28}}^4\left[\begin{array}{c} a_{1,}, a_{2,}, a_{1,}, a_{2,} b_{1,} b_{2,}, b_{1,} b_{2,} c_{1,} c_{1,} c_{2,} c_{3;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ , \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{array}\right]\ldots\ldots\ldots\ldots\ldots$$

…………………………………………………………..(1.2.28)

Where

${F_{28}}^4$ $[a_{1,}, a_{2,}, a_{1,}, a_{2,} b_{1,} b_{2,}, b_{1,} b_{2,} c_{1,} c_{2,} c_{1,} c_{3;,} X, Y, Z, T]$

$$=\sum_{p,q,r,s=0}^{\infty} \frac{(a_{1,})_{p+r}(a_{2,})_{q+s}(b_{1,})_{p+r}(b_{2,})_{q+s}}{p!s!q!r!(c_{1,})_{p+q}(C_{2,})_{p+q}(C_{3,})_s} \ X^p \ Y^q \ Z^r \ T^s$$

Then probability density function (p.d.f) of (1.2.28) is given by :

$$F(U) = \frac{U^{a-\frac{m+1}{2}} \ (I-U)^{b-\frac{m+1}{2}} \ F_1(X_1){F_{28}}^4 [X_2]}{L{F_{28}}^4 [X_{30}]}$$

=0 elsewhere

Where

$X_{30} =$

$$\left[\begin{array}{c} a_{1,}, a_{2,}, a_{1}, a_{2,} b_{1,} b_{2,}, b_{1,} b_{2,} c_{1,} c_{1,} c_{2,} c_{3;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ \Delta\left(\frac{n}{2}, \frac{1+c+b}{2}\right), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{array}\right]$$

[29] $\int_0^I U^{a-\frac{m+1}{2}} \ (I-U)^{b-\frac{m+1}{2}} \ F_1$ (a,1-a;c;$\frac{U}{2}$).

${F_{29}}^4$ [$(I-U)^n$ X,$(I-U)^n$ Y, $(I-U)^n$ Z, $(I-U)^n$T]dU

$$=LF_{29}{}^{4}\left[\begin{matrix} a_{1},a_{1},a_{2},a_{2},b_{1},b_{2},b_{1},b_{2},c_{1},c_{1},c_{3},c_{1;},\Delta(n,b),\Delta(\frac{n}{2},\frac{c+b}{2}), \\ ,\Delta(\frac{n}{2},\frac{1+c+b}{2}),\Delta(n,c+b),\Delta\left(\frac{n}{2},c+b+a\right),\Delta(\frac{n}{2},\frac{1+c+b-a}{2});X;Y;Z;T \end{matrix}\right]\ldots\ldots\ldots\ldots\ldots$$

……………………………………………………..(1.2.29)

Where

$F_{28}{}^{4}\,[a_{1},a_{1},a_{2},a_{2},b_{1},b_{2},b_{1},b_{2},c_{1},c_{2},c_{3},c_{1;},X,Y,Z,T]$

$$=\sum_{p,q,r,s=0}^{\infty}\frac{(a_{1})_{p+q}(a_{2})_{r+s}(b_{1})_{p+r}(b_{2})_{q+s}}{p!s!q!r!(c_{1})_{p+s}(C_{2})_{q}(C_{3})_{r}}\;X^{p}\;Y^{q}\;Z^{r}\;T^{s}$$

Then probability density function (p.d.f) of (1.2.29) is given by :

$$F(U)=\frac{U^{a-\frac{m+1}{2}}\;(I-U)^{b-\frac{m+1}{2}}\;F_{1}(X_{1})F_{29}{}^{4}\,[X_{2}]}{LF_{28}{}^{4}\,[X_{31}]}$$

=0 elsewhere

Where

$X_{31}=$

$$\left[\begin{matrix} a_{1},a_{1},a_{2},a_{2},b_{1},b_{2},b_{1},b_{2},c_{1},c_{2},c_{3},c_{1;},\Delta(n,b),\Delta(\frac{n}{2},\frac{c+b}{2}), \\ \Delta\left(\frac{n}{2},\frac{1+c+b}{2}\right),\Delta(n,c+b),\Delta\left(\frac{n}{2},c+b+a\right),\Delta(\frac{n}{2},\frac{1+c+b-a}{2});X;Y;Z;T \end{matrix}\right]$$

[30] $\int_{0}^{I}U^{a-\frac{m+1}{2}}\;(I-U)^{b-\frac{m+1}{2}}\;F_{1}$ (a,1-a;c;$\frac{U}{2}$).

$F_{30}{}^{4}\,[(I-U)^{n}$ X,$(I-U)^{n}$ Y, $(I-U)^{n}$ Z, $(I-U)^{n}$T]dU

$$=LF_{30}{}^{4}\left[\begin{matrix} a_{1},a_{1},a_{2},a_{2},b_{1},b_{2},b_{3},b_{1},c_{1},c_{1},c_{2},c_{3;},\Delta(n,b),\Delta(\frac{n}{2},\frac{c+b}{2}), \\ ,\Delta(\frac{n}{2},\frac{1+c+b}{2}),\Delta(n,c+b),\Delta\left(\frac{n}{2},c+b+a\right),\Delta(\frac{n}{2},\frac{1+c+b-a}{2});X;Y;Z;T \end{matrix}\right]\ldots\ldots\ldots\ldots\ldots$$

…………..…………………………………………..(1.1.30)

Where

$F_{30}{}^{4}\,[a_{1},a_{1},a_{2},a_{2},b_{1},b_{1},b_{2},b_{3},c_{1},c_{2},c_{1},c_{3;},X,Y,Z,T]$

$$=\sum_{p,q,r,s=0}^{\infty}\frac{(a_{1})_{p+q}(a_{2})_{r+s}(b_{1})_{p+q}(b_{2})_{s}}{p!s!q!r!(c_{1})_{p+r}(C_{2})_{q}(C_{3})_{s}}\;X^{p}\;Y^{q}\;Z^{r}\;T^{s}$$

Then probability density function (p.d.f) of (1.1.30) is given by :

$$F(U) = \frac{U^{a-\frac{m+1}{2}} \ (I-U)^{b-\frac{m+1}{2}} \ F_1(X_1) F_{30}{}^4 \, [X_2]}{LF_{30}{}^4 \, [X_{32}]}$$

=0 elsewhere

Where

$X_{32} =$

$$\left[\begin{array}{c} a_{1,}, a_{1,}, a_2, a_{2,} b_{1,} b_{2,}, b_{3,} b_{1,} c_{1,} c_{1,} c_{2,} c_{3;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ \Delta\left(\frac{n}{2}, \frac{1+c+b}{2}\right), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{array}\right]$$

[31] $\int_0^I U^{a-\frac{m+1}{2}} \ (I-U)^{b-\frac{m+1}{2}} \ F_1$ (a,1-a;c;$\frac{U}{2}$).

$F_{31}{}^4 \, [(I-U)^n$ X,$(I-U)^n$ Y, $(I-U)^n$ Z, $(I-U)^n$T]dU

$$= LF_{31}{}^4 \left[\begin{array}{c} a_{1,}, a_{1,}, a_{2,}, a_{2,} b_{1,} b_{2,}, b_{3,} b_{1,} c_{1,} c_{1,} c_{2,} c_{3;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ , \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{array}\right] \ldots\ldots\ldots\ldots\ldots$$

……………………………………………………………..(1.1.31)

Where

$F_{31}{}^4 \, [a_{1,}, a_{1,}, a_{2,}, a_{2,} b_{1,} b_{2,}, b_{1,} b_{3,} c_{1,} c_{1,} c_{2,} c_{3;,} X, Y, Z, T]$

$$= \sum_{p,q,r,s=0}^{\infty} \frac{(a_{1,})_{p+q} (a_{2,})_{r+s} (b_{1,})_{p+q} (b_{2,})_{r+s}}{p!s!q!r!(c_{1,})_{p+q(C_{2,})_r} (C_{3,})_s} \ X^p \ Y^q \ Z^r \ T^s$$

Then probability density function (p.d.f) of (1.1.31) is given by :

$$F(U) = \frac{U^{a-\frac{m+1}{2}} \ (I-U)^{b-\frac{m+1}{2}} \ F_1(X_1) F_{31}{}^4 \, [X_2]}{LF_{31}{}^4 \, [X_{33}]}$$

=0 elsewhere

Where

$X_{33} =$

$$\left[\begin{array}{c} a_{1,}, a_{1,}, a_2, a_{2,} b_{1,} b_{2,}, b_{3,} b_{1,} c_{1,} c_{1,} c_{2,} c_{3;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ \Delta\left(\frac{n}{2}, \frac{1+c+b}{2}\right), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{array}\right]$$

[32] $\int_0^I U^{a-\frac{m+1}{2}} \; (I-U)^{b-\frac{m+1}{2}} \; F_1$ (a,1-a;c;$\frac{U}{2}$).

${F_{32}}^4$ $[(I-U)^n$ X,$(I-U)^n$ Y, $(I-U)^n$ Z, $(I-U)^n$T]dU

$$=L{F_{32}}^4\left[\begin{array}{c} a_{1,}, a_{1,}, a_{2,}, a_{2,} b_{1,} b_{2,}, b_{3,} b_{1,} c_{1,} c_{1,} c_{3,} c_{1;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ , \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{array}\right]\ldots\ldots\ldots\ldots\ldots$$

……………………………………………………..(1.1.32)

Where

${F_{32}}^4$ $[a_{1,}, a_{1,}, a_{2,}, a_{2,} b_{1,} b_{2,}, b_{1,} b_{3,} c_{1,} c_{1,} c_{3,} c_{1;,} X, Y, Z, T]$

$$=\sum_{p,q,r,s=0}^{\infty} \frac{(a_{1,})_{p+q} (a_{2,})_r (b_{1,})_s (b_{2,})_{p+q+r+s}}{p!s!q!r!(c_{1,})_{p}(c_{2,})_q\,(c_{3,})_{r+s}} \quad X^p \;\; Y^q \; Z^r \; T^s$$

Then probability density function (p.d.f) of (1.1.32) is given by :

$$F(U) = \frac{U^{a-\frac{m+1}{2}} \; (I-U)^{b-\frac{m+1}{2}} \; F_1(X_1) {F_{32}}^4 \, [X_2]}{L{F_{32}}^4 \, [X_{34}]}$$

=0 elsewhere

Where

$X_{34} =$

$$\left[\begin{array}{c} a_{1,}, a_{1,}, a_2, a_{2,} b_{1,} b_{2,}, b_{1,} b_{3,} c_{1,} c_{2,} c_{3,} c_{1;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ \Delta\left(\frac{n}{2}, \frac{1+c+b}{2}\right), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{array}\right]$$

[33] $\int_0^I U^{a-\frac{m+1}{2}} \; (I-U)^{b-\frac{m+1}{2}} \; F_1$ (a,1-a;c;$\frac{U}{2}$).

${F_{33}}^4$ $[(I-U)^n$ X,$(I-U)^n$ Y, $(I-U)^n$ Z, $(I-U)^n$T]dU

$$=LF_{33}{}^{4}\left[\begin{array}{c} a_{1}, a_{1}, a_{2}, a_{2}, b_{1}, b_{2}, b_{1}, b_{3}, c_{1}, c_{2}, c_{3}, c_{2;}, \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ , \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{array}\right]\ldots\ldots\ldots\ldots\ldots$$

………..…………………………………………..(1.1.33)

Where

$F_{33}{}^{4}\ [a_{1}, a_{1}, a_{2}, a_{2}, b_{1}, b_{2}, b_{1}, b_{3}, c_{1}, c_{2}, c_{3}, c_{2;}, X, Y, Z, T]$

$$=\sum_{p,q,r,s=0}^{\infty} \frac{(a_{1})_{p+q}(a_{2})_{r+s}(b_{1})_{p+r}(b_{2})_{q}(b_{3})_{s}}{p!s!q!r!(c_{1})_{p(C_{2})_{q+s}}(C_{3})_{r}} \quad X^{p}\ \ Y^{q}\ Z^{r}\ T^{s}$$

Then probability density function (p.d.f) of (1.1.33) is given by :

$$F(U) = \frac{U^{a-\frac{m+1}{2}}\ (I-U)^{b-\frac{m+1}{2}}\ F_{1}(X_{1}) F_{33}{}^{4}\ [X_{2}]}{LF_{33}{}^{4}\ [X_{35}]}$$

=0 elsewhere

Where

$X_{35} =$

$$\left[\begin{array}{c} a_{1}, a_{1}, a_{2}, a_{2}, b_{1}, b_{2}, b_{1}, b_{3}, c_{1}, c_{2}, c_{3}, c_{2;}, \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ \Delta\left(\frac{n}{2}, \frac{1+c+b}{2}\right), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{array}\right]$$

[34] $\int_{0}^{I} U^{a-\frac{m+1}{2}}\ (I-U)^{b-\frac{m+1}{2}}\ F_{1}$ (a,1-a;c;$\frac{U}{2}$).

$F_{34}{}^{4}$ [$(I-U)^{n}$ X,$(I-U)^{n}$ Y, $(I-U)^{n}$ Z, $(I-U)^{n}$T]dU

$$=LF_{34}{}^{4}\left[\begin{array}{c} a_{1}, a_{1}, a_{2}, a_{2}, b_{1}, b_{2}, b_{1}, b_{3}, c_{1}, c_{2}, c_{1}, c_{3;}, \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ , \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{array}\right]\ldots\ldots\ldots\ldots\ldots$$

……………..……………………………………..(1.1.34)

Where

$F_{34}{}^{4}\ [a_{1}, a_{1}, a_{2}, a_{2}, b_{1}, b_{2}, b_{1}, b_{3}, c_{1}, c_{2}, c_{1}, c_{3;}, X, Y, Z, T]$

$$=\sum_{p,q,r,s=0}^{\infty} \frac{(a_{1})_{p+q}(a_{2})_{r+s}(b_{1})_{p+r}(b_{2})_{q}(b_{3})_{s}}{p!s!q!r!(c_{1})_{p+r}(C_{2})_{p+r}(C_{3})_{s}} \quad X^{p}\ \ Y^{q}\ Z^{r}\ T^{s}$$

Then probability density function (p.d.f) of (1.1.34) is given by :

$$F(U) = \frac{U^{a-\frac{m+1}{2}}\ (I-U)^{b-\frac{m+1}{2}}\ F_1(X_1) {F_{34}}^4\,[X_2]}{L{F_{34}}^4\,[X_{36}]}$$

=0 elsewhere

Where

$X_{36} =$

$$\begin{bmatrix} a_{1\,,}, a_{1\,,}, a_2\,, a_{2\,,} b_{1\,,} b_{2,}, b_{1,} b_{3\,,} c_{1\,,} c_{2\,,} c_{1\,,} c_{3;\,,} \Delta(n,b), \Delta(\frac{n}{2}\,, \frac{c+b}{2})\,, \\ \Delta\left(\frac{n}{2}\,, \frac{1+c+b}{2}\right), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}\,, \frac{1+c+b-a}{2}); X; Y; Z; T \end{bmatrix}$$

[35] $\int_0^I U^{a-\frac{m+1}{2}}\ (I-U)^{b-\frac{m+1}{2}}\ F_1$ (a,1-a;c;$\frac{U}{2}$).

${F_{35}}^4$ [$(I-U)^n$ X,$(I-U)^n$ Y, $(I-U)^n$ Z, $(I-U)^n$T]dU

$$=L{F_{35}}^4 \begin{bmatrix} a_{1\,,}, a_{1\,,}, a_{2\,,}, a_{2\,,} b_{1\,,} b_{2\,,}, b_{3\,,} b_{4\,,} c_{1\,,} c_{2,} c_{1,} c_{3;\,,} \Delta(n,b), \Delta(\frac{n}{2}\,, \frac{c+b}{2})\,, \\ , \Delta(\frac{n}{2}\,, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}\,, \frac{1+c+b-a}{2}); X; Y; Z; T \end{bmatrix} \ldots\ldots\ldots\ldots\ldots$$

…………..…………………………………………..(1.1.35)

Where

${F_{35}}^4\,[a_{1\,,}, a_{1\,,}, a_{2\,,}, a_{2\,,} b_{1\,,} b_{2\,,}, b_{3\,,} b_{4\,,} c_{1\,,} c_{2\,,} c_{1\,,} c_{3;\,,} X, Y, Z, T]$

$$=\sum_{p,q,r,s=0}^{\infty} \frac{(a_{1\,,})_{p+q} (a_{2\,,})_{r+s} (b_{1\,,})_p (b_{2\,,})_q (b_{3\,,})_s}{p!s!q!r!(c_{1,})_{p+r} (C_{2\,,})_q\ (C_{3\,,})_s}\ \ X^p\ \ Y^q\ Z^r\ T^s$$

Then probability density function (p.d.f) of (1.1.35) is given by :

$$F(U) = \frac{U^{a-\frac{m+1}{2}}\ (I-U)^{b-\frac{m+1}{2}}\ F_1(X_1) {F_{35}}^4\,[X_2]}{L{F_{35}}^4\,[X_{37}]}$$

=0 elsewhere

Where

$X_{37} =$

$$\begin{bmatrix} a_{1,}, a_{1,}, a_{2}, a_{2,} b_{1,} b_{2,}, b_{3,} b_{4,} c_{1,} c_{2,} c_{1,} c_{3;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ \Delta\left(\frac{n}{2}, \frac{1+c+b}{2}\right), \Delta(n,c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{bmatrix}$$

[36] $\int_0^I U^{a-\frac{m+1}{2}} \ (I-U)^{b-\frac{m+1}{2}} \ F_1$ (a,1-a;c;$\frac{U}{2}$).

${F_{36}}^4$ [$(I-U)^n$ X,$(I-U)^n$ Y, $(I-U)^n$ Z, $(I-U)^n$T]dU

$$=L{F_{36}}^4 \begin{bmatrix} a_{1,}, a_{1,}, a_{2,}, a_{3,} b_{1,} b_{2,}, b_{1,} b_{3,} c_{1,} c_{2,} c_{3,} c_{1;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ , \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n,c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{bmatrix} \ldots\ldots\ldots\ldots\ldots$$

………………………………………………………..(1.1.36)

Where

${F_{36}}^4$ $[a_{1,}, a_{1,}, a_{2,}, a_{3,} b_{1,} b_{2,}, b_{1,} b_{3,} c_{1,} c_{2,} c_{3,} c_{1;,} X, Y, Z, T]$

$$=\sum_{p,q,r,s=0}^{\infty} \frac{(a_{1,})_{p+q}(a_{2,})_r(a_{3,})_s(b_{1,})_{p+r}(b_{2,})_q(b_{3,})_s}{p!s!q!r!(c_{1,})_{p+s(C_{2,})_q}\,(C_{3,})_r} \quad X^p \ Y^q \ Z^r \ T^s$$

Then probability density function (p.d.f) of (1.1.36) is given by :

$$\text{F(U)} = \frac{U^{a-\frac{m+1}{2}} \ (I-U)^{b-\frac{m+1}{2}} \ F_1(X_1){F_{36}}^4 \,[X_2]}{L{F_{36}}^4 \,[X_{38}]}$$

=0 elsewhere

Where

$X_{38} =$

$$\begin{bmatrix} a_{1,}, a_{1,}, a_{2}, a_{3,} b_{1,} b_{2,}, b_{1,} b_{3,} c_{1,} c_{2,} c_{3,} c_{1;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ \Delta\left(\frac{n}{2}, \frac{1+c+b}{2}\right), \Delta(n,c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{bmatrix}$$

[37] $\int_0^I U^{a-\frac{m+1}{2}} \ (I-U)^{b-\frac{m+1}{2}} \ F_1$ (a,1-a;c;$\frac{U}{2}$).

${F_{37}}^4$ [$(I-U)^n$ X,$(I-U)^n$ Y, $(I-U)^n$ Z, $(I-U)^n$T]dU

$$=LF_{37}{}^{4}\begin{bmatrix} a_{1,}, a_{1,}, a_{2,}, a_{3,} b_{1,} b_{2,}, b_{1,} b_{3,} c_{1,} c_{2,} c_{3,} c_{3;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ , \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); \mathrm{X; Y; Z; T} \end{bmatrix} \ldots\ldots\ldots\ldots\ldots$$

……………………………………………………..(1.1.37)

Where

$F_{37}{}^{4}\ [a_{1,}, a_{1,}, a_{2,}, a_{3,} b_{1,} b_{2,}, b_{1,} b_{3,} c_{1,} c_{2,} c_{3,} c_{3;,} X, Y, Z, T]$

$$=\sum_{p,q,r,s=0}^{\infty} \frac{(a_{1,})_{p+q}(a_{2,})_{r}(a_{3,})_{s}(b_{1,})_{p+r}(b_{2,})_{q}(b_{3,})_{s}}{p!s!q!r!(c_{1,})_{p(C_{2,})_{q}}(C_{3,})_{r+s}} \quad X^{p}\ \ Y^{q}\ Z^{r}\ T^{s}$$

Then probability density function (p.d.f) of (1.1.37) is given by :

$$\mathrm{F(U)} = \frac{U^{a-\frac{m+1}{2}}\ (I-U)^{b-\frac{m+1}{2}}\ F_{1}(X_{1}) F_{37}{}^{4}\ [X_{2}]}{LF_{37}{}^{4}\ [X_{39}]}$$

=0 elsewhere

Where

$X_{39} =$

$$\begin{bmatrix} a_{1,}, a_{1,}, a_{2}, a_{3,} b_{1,} b_{2,}, b_{1,} b_{3,} c_{1,} c_{2,} c_{3,} c_{3;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ \Delta\left(\frac{n}{2}, \frac{1+c+b}{2}\right), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); \mathrm{X; Y; Z; T} \end{bmatrix}$$

[38] $\int_{0}^{I} U^{a-\frac{m+1}{2}}\ (I-U)^{b-\frac{m+1}{2}}\ F_{1}$ (a,1-a;c;$\frac{U}{2}$).

$F_{38}{}^{4}\ [(I-U)^{n}$ X,$(I-U)^{n}$ Y, $(I-U)^{n}$ Z, $(I-U)^{n}$T]dU

$$=LF_{38}{}^{4}\begin{bmatrix} a_{1,}, a_{1,}, a_{2,}, a_{2,} b_{1,} b_{2,}, b_{1,} b_{3,} c_{1,} c_{2,} c_{1,} c_{2;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ , \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); \mathrm{X; Y; Z; T} \end{bmatrix} \ldots\ldots\ldots\ldots\ldots$$

……………..……………………………………..(1.1.38)

Where

$F_{38}{}^{4}\ [a_{1,}, a_{1,}, a_{2,}, a_{2,} b_{1,} b_{1,}, b_{1,} b_{1,} c_{1,} c_{2,} c_{1,} c_{2;,} X, Y, Z, T]$

$$=\sum_{p,q,r,s=0}^{\infty} \frac{(a_{1,})_{p+q}(a_{2,})_{r+s}(b_{1,})_{p+q+r+s}}{p!s!q!r!(c_{1,})_{p+r(C_{2,})_{q+s}}} \quad X^{p}\ \ Y^{q}\ Z^{r}\ T^{s}$$

Then probability density function (p.d.f) of (1.1.38) is given by :

$$\mathrm{F(U)} = \frac{U^{a-\frac{m+1}{2}} \ (I-U)^{b-\frac{m+1}{2}} \ F_1(X_1) {F_{38}}^4 \, [X_2]}{L{F_{38}}^4 \, [X_{40}]}$$

=0 elsewhere

Where

$X_{40} =$

$$\left[\begin{array}{c} a_{1\,,}, a_{1\,,}, a_{2}\,, a_{2\,,} b_{1\,,} b_{1,}, b_{1,} b_{1\,,} c_{1\,,} c_{2\,,} c_{1\,,} c_{2;\,,} \Delta(n,b), \Delta(\frac{n}{2}\,, \frac{c+b}{2})\,, \\ \Delta\left(\frac{n}{2}\,, \frac{1+c+b}{2}\right), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}\,, \frac{1+c+b-a}{2}); \mathrm{X; Y; Z; T} \end{array}\right]$$

[39] $\int_0^I U^{a-\frac{m+1}{2}} \ (I-U)^{b-\frac{m+1}{2}} \ F_1$ (a,1-a;c;$\frac{U}{2}$).

${F_{39}}^4$ [$(I-U)^n$ X,$(I-U)^n$ Y, $(I-U)^n$ Z, $(I-U)^n$T]dU

$$= L{F_{39}}^4 \left[\begin{array}{c} a_{1\,,}, a_{1\,,}, a_{2\,,}, a_{3\,,} b_{1\,,} b_{1\,,}, b_{1\,,} b_{1\,,} c_{1\,,} c_{2,} c_{1,} c_{2;\,,} \Delta(n,b), \Delta(\frac{n}{2}\,, \frac{c+b}{2})\,, \\ , \Delta(\frac{n}{2}\,, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}\,, \frac{1+c+b-a}{2}); \mathrm{X; Y; Z; T} \end{array}\right] \ldots\ldots\ldots\ldots\ldots$$

…………………………………………………..(1.1.39)

Where

${F_{39}}^4$ $[a_{1\,,}, a_{1\,,}, a_{2\,,}, a_{3\,,} b_{1\,,} b_{1\,,}, b_{1\,,} b_{1\,,} c_{1\,,} c_{2\,,} c_{1\,,} c_{2;\,,} X, Y, Z, T]$

$$= \sum_{p,q,r,s=0}^{\infty} \frac{(a_{1\,,})_{p+q} (a_{2\,,})_r (a_{3\,,})_s (b_{1\,,})_{p+q+r+s}}{p!s!q!r!(c_{1,})_{p+r} (c_{2\,,})_{q+s}} \ \ X^p \ \ Y^q \ Z^r \ T^s$$

Then probability density function (p.d.f) of (1.1.39) is given by :

$$\mathrm{F(U)} = \frac{U^{a-\frac{m+1}{2}} \ (I-U)^{b-\frac{m+1}{2}} \ F_1(X_1) {F_{39}}^4 \, [X_2]}{L{F_{39}}^4 \, [X_{41}]}$$

=0 elsewhere

Where

$X_{41} =$

$$\left[\begin{array}{c} a_{1,}, a_{1,}, a_2, a_{3,} b_{1,} b_{1,}, b_{1,} b_{1,} c_{1,} c_{2,} c_{1,} c_{2;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ \Delta\left(\frac{n}{2}, \frac{1+c+b}{2}\right), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); \text{X; Y; Z; T} \end{array}\right]$$

[40] $\int_0^I U^{a-\frac{m+1}{2}} \ (I-U)^{b-\frac{m+1}{2}} \ F_1$ (a,1-a;c;$\frac{U}{2}$).

${F_{40}}^4$ [$(I-U)^n$ X,$(I-U)^n$ Y, $(I-U)^n$ Z, $(I-U)^n$T]dU

$$=L{F_{40}}^4\left[\begin{array}{c} a_{1,}, a_{2,}, a_{3,}, a_{4,} b_{1,} b_{1,}, b_{1,} b_{1,} c_{1,} c_{1,} c_{2,} c_{2;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ , \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); \text{X; Y; Z; T} \end{array}\right]\ldots\ldots\ldots\ldots\ldots$$

……………..……………………………………..(1.1.40)

Where

${F_{40}}^4$ $[a_{1,}, a_{2,}, a_{3,}, a_{4,} b_{1,} b_{1,}, b_{1,} b_{1,} c_{1,} c_{1,} c_{2,} c_{2;,} X, Y, Z, T]$

$$=\sum_{p,q,r,s=0}^{\infty} \frac{(a_{1,})_p (a_{2,})_q (a_{3,})_r (a_{4,})_s (b_{1,})_{p+q+r+s}}{p!s!q!r!(c_{1,})_{p+q}(c_{2,})_{r+s}} \ X^p \ Y^q \ Z^r \ T^s$$

Then probability density function (p.d.f) of (1.1.40) is given by :

$$\text{F(U)} = \frac{U^{a-\frac{m+1}{2}} \ (I-U)^{b-\frac{m+1}{2}} \ F_1(X_1) {F_{40}}^4 \, [X_2]}{L{F_{40}}^4 \, [X_{42}]}$$

=0 elsewhere

Where

$X_{42} =$

$$\left[\begin{array}{c} a_{1,}, a_{2,}, a_{3}, a_{4,} b_{1,} b_{1,}, b_{1,} b_{1,} c_{1,} c_{1,} c_{2,} c_{2;,} \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ \Delta\left(\frac{n}{2}, \frac{1+c+b}{2}\right), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); \text{X; Y; Z; T} \end{array}\right]$$

1.3 SOLUTION OF INTRGRALS

One of the proof is , expressing the quadruple hypergeometric function in terms of equivalent series , in the integrand of the (2.1.1) . we find that the integral becomes .

$\int_0^I U^{a-\frac{m+1}{2}} \ (I-U)^{b-\frac{m+1}{2}} \ F_1$ (a,1-a;c;$\frac{U}{2}$).

$F_1{}^4$ $[a_{1,}, a_{1,}, a_{1,}, a_{2,} b_{1,} b_{1,}, b_{1,} b_{1,} c_{1,} c_{2,} c_{3,} c_{4;,} (I-U)^n$ X,$(I-U)^n$ Y, $(I-U)^n$ Z, $(I-U)^n$T]dU

Or

$\int_0^I U^{a-\frac{m+1}{2}}\ (I-U)^{b-\frac{m+1}{2}}\ F_1$ (a,1-a;c;$\frac{U}{2}$)………………..………(1.3.1)

$\sum_{p,q,r,s=0}^{\infty} A_{r,s}{}^{p,q}$ $[(I-U)^n$ X,$(I-U)^n$ Y, $(I-U)^n$ Z, $(I-U)^n$T]dU

Where $A_{r,s}{}^{p,q}$ stands for the expression

$\sum_{p,q,r,s=0}^{\infty} \frac{(a_{1,})_{p+q+r}(a_{2,})_s(b_{1,})_{p+q+r+s}}{p!s!q!r!(c_{1,})_p(C_{2,})_q(c_{3,})_r(C_{4,})_s}$ $[(I-U)X]^p[(I-U)Y]^q[\ (I-U)Z]^r\ [\ (I-U)^s]$

We assume that the series is uniformly convergent in the region of integration , the inversion of integration and summation is infinite, then integral .

$=\int_0^I U^{a-\frac{m+1}{2}}\ (I-U)^{b-\frac{m+1}{2}}\ F_1$ (a,1-a;c;$\frac{U}{2}$)…………………………(1.3.2)

$A_{r,s}{}^{p,q}[X^p(I-U)^{pn}\ Y^q,(I-U)^{qn}\ \ Z^T(I-U)^m\ \ T^s(I-U)^{sn}$T]dU

$\sum_{p,q,r,s=0}^{\infty} A_{r,s}{}^{p,q}X^p\ \ Y^q\ Z^r\ T^s\ \int_0^I U^{a-\frac{m+1}{2}}\ F_1$ (a,1-a;c;$\frac{U}{2}$)

$[(I-U)^{n(p+q+r+s)}]$dU$(I-U)^{b-\frac{m+1}{2}}$

$$\sum_{p,q,r,s=0}^{\infty} A_{r,s}{}^{p,q}X^p\ \ Y^q\ Z^r\ T^s\ \int_0^I U^{a-\frac{m+1}{2}}\ (I-U)^{b+n(p+q+r+s)-\frac{m+1}{2}}$$

F_1 (a,1-a;c;$\frac{U}{2}$) dU

On evaluating the integral by means of the formula

$\int_0^I X^{a-\frac{m+1}{2}}\ (I-X)^{b-\frac{m+1}{2}}\ F_1$ (a,1-a;c;$\frac{U}{2}$)dX

$=\frac{\Gamma_m(C)\Gamma_m(b)\Gamma_m(\frac{c+b}{2})\Gamma_m(\frac{1+c+b}{2})}{\Gamma_m(C+b)\Gamma_m(\frac{c+b+a}{2})\Gamma_m(\frac{1+c+b-a}{2})}$………………………………..(1.3.3)

Where Re (a), Re (b)>0

We see that the value of the integral is

$\sum_{p,q,r,s=0}^{\infty} A_{r,s}{}^{p,q} X^p\ Y^q\ Z^r\ T^s\ \frac{\Gamma_m(C)\Gamma_m(b+n(p+q+r+s))\Gamma_m(\frac{c+b+n(p+q+r+s)}{2})}{\Gamma_m(C+b+n(p+q+r+s))\Gamma_m[\frac{c+b+n(p+q+r+s)-a}{2}]}$

$\frac{\Gamma_m(\frac{c+b+n(p+q+r+s)}{2})}{\Gamma_m[\frac{c+b+n(p+q+r+s)-a}{2}]}$…………………………………..………… (1.3.4)

$$\sum_{p,q,r,s=0}^{\infty} A_{r,s}{}^{p,q} X^p\ Y^q\ Z^r\ T^s\ \frac{\Gamma_m(C)\Gamma_m(b)\Gamma_m(\frac{c+b}{2})\Gamma_m(\frac{1+c+b}{2})}{\Gamma_m(C+b)\Gamma_m(\frac{c+b+a}{2})\Gamma_m(\frac{1+c+b-a}{2})}$$

$$\frac{(b)_{n(p+q+r+s)}(\frac{c+b}{2})_{n(p+q+r+s)}(\frac{1+c+b}{2})_{n(p+q+r+s)}}{(c+b)_{n(p+q+r+s)}(\frac{c+b+a}{2})_{\frac{n}{2}(p+q+r+s)}(\frac{1+c+b-a}{2})_{\frac{n}{2}(p+q+r+s)}}$$

$= L\sum_{p,q,r,s=0}^{\infty} \frac{(a_1,)_{p+q+r}(a_2,)_s(b_1,)_{p+q+r+s}}{p!s!q!r!(c_1,)_{p}(c_2,)_q\ (C_3,)_r(C_4,)_s}$

$$\frac{(b)_{n(p+q+r+s)}(\frac{c+b}{2})_{n(p+q+r+s)}(\frac{1+c+b}{2})_{n(p+q+r+s)}}{(c+b)_{n(p+q+r+s)}(\frac{c+b+a}{2})_{\frac{n}{2}(p+q+r+s)}(\frac{1+c+b-a}{2})_{\frac{n}{2}(p+q+r+s)}} X^pY^qZ^rT^s$$

Now if we apply the formula

$(\alpha)_{k1} = k^{k1} \prod_{j=1}^{k}\ \ \{\frac{(\alpha+j-1)}{k}\}$

Where k is positive integer and non negative , there after little simplification arrive at the result (1) is

$=LF_1{}^4\left[\begin{matrix} a_1,, a_1,, a_1,, a_2, b_1, b_1,, b_1, b_1, c_1, c_2, c_3, c_4;, \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ , \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta\left(\frac{n}{2}, c+b+a\right), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X; Y; Z; T \end{matrix}\right]$………………

……………………………………….…………..(1.3.5)

Proof of the integral from (1.2.2) to (1.1.40) is similar. Therefore forty direct result have quoted.

Conclusion

In this paper we have evaluated forty integrals associated with Hypergeometric function of four variable of matrix argument with their statistical distribution. All the matrices involved are real positive definite symmetric of order mxm.

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